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The transient transmission through a quantum dot under the influence of oscillating external fields

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Abstract. We consider a quantum dot coupled to two leads in the presence of oscillating external fields applied to the dot and the two leads. Using the nonequilibrium-Green-function method, the formula for the transient transmission probability (TTP) $T_{\varepsilon_f,\varepsilon_i}(t',t)$ is derived. The numerical studies reveal the following facts. When $t'-t \sim 1/\Gamma$ (Γ is the full width of the resonant state), no visible resonant tunnelling behaviour occurs, and $T_{\varepsilon_f,\varepsilon_i}(t',t)$ strongly depends on the time at which the electron enters the dot, t, and that at which it exits from it, t'; when $t'-t \gg 1/\Gamma$, the resonant tunnelling becomes striking, and $T_{\varepsilon_f,\varepsilon_i}(t',t)$ depends only weakly on t and t'. The conventional transmission probability $T_{\varepsilon_f,\varepsilon_i}$, defined as the limit as $t \to -\infty$ and $t' \to +\infty$, has a very rich spectrum and exhibits a simultaneous quenching of some channels. On the basis of the properties of $T_{\varepsilon_f,\varepsilon_i}$, the condition for the occurrence of an electron–photon pump has also been discussed.

1. Introduction

Recently, the time-dependent transport phenomena in mesoscopic systems have been attracting more and more attention. One of the interesting problems is that of studying the effects on the probability of electron transmission through a nanostructure of oscillating external fields. Theoretically, Sokolovski investigated the resonance tunnelling through a quantum well in the presence of a harmonic external field, and showed that the resonance peak split into a family of peaks with the spacing ω (the frequency of the external field, in units in which $\hbar = 1$) [1]. Wagner studied the electron resonant transmission through an oscillating quantum well, and found a rich spectrum of sidebands and a strong simultaneous reduction at a certain characteristic ratio V/ω [2, 3]. Johansson and Wendin [4] investigated the probability of transmission through an irradiated double-barrier structure (DBS) with two different resonant levels in the quantum well. Iñarrea *et al* [5, 6] used the second-quantized method to deal with an external electromagnetic field. Yakubo *et al* [7] investigated the necessary conditions for photon-assisted tunnelling, etc, to have a strong influence. Experimentally, a number of new phenomena have been observed, such as the electron–photon pump, by Kouwenhoven *et al* [8, 9], and various photon-assisted tunnelling peaks, by Blick *et al* [10] and Drexler *et al* [11]. In most of the previous work, only the conventional transmission probability $T_{\varepsilon_f,\varepsilon_i}$ was studied [1–4, 12, 13], where $T_{\varepsilon_f,\varepsilon_i}$ is defined as the limit

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of the transient transmission probability (TTP), $T_{\varepsilon_f,\varepsilon_i}(t',t)$, for $t \to -\infty$ and $t' \to +\infty$; here t and t' are the times at which the electron enters and leaves the dot, respectively. However, since the electron needs a certain time to tunnel through a DBS or to develop resonant tunnelling behaviour, studying the time dependence behaviour of the TTP with t and t' at finite values is of interest; this is the main goal of this work.

The system under consideration is a quantum dot coupled to two leads connected to reservoirs. The harmonic external fields are applied to three regions of the system (the left-hand lead, the right-hand lead, and the quantum dot), respectively. We assume that the time-dependent external fields only change the single-electron energies without changing their occupations in the same region (the adiabatic approximation) [14, 15]; moreover, the effect of the magnetic fields can be neglected.

In this paper, we adopt the nonequilibrium-Green-function (NGF) method to derive the general formula for the TTP, $T_{\varepsilon_f,\varepsilon_i}(t',t)$. Compared with the approach of directly solving the Schrödinger equation, two advantages are obvious: a batch of well established techniques can be used for the calculation, and more statistical information is obtained. (In particular, if a many-body effect, such as the intra-dot electron–electron Coulomb interaction of the quantum dot system, is being considered, one can calculate the many-body effect to all orders in the perturbation by the NGF method.) From the numerical studies we establish the following facts. When $t'-t \sim 1/\Gamma$, the resonant tunnelling is not visible. A certain proportion of the transmission probability will be allocated to the electronic states with $\varepsilon_f - \varepsilon_i \neq n\omega$ ($n = 0, \pm 1, \pm 2, \ldots$), and $T_{\varepsilon_f,\varepsilon_i}(t',t)$ is strongly dependent on both the time t at which the electron enters the dot and that, t', at which it leaves the dot. On the other hand, when $t'-t \gg 1/\Gamma$, the resonant behaviour is striking, and $T_{\varepsilon_f,\varepsilon_i}(t',t)$ depends only weakly on t' and t. The condition for photon-assisted resonant tunnelling is $t'-t \gg 1/\Gamma > 1/\omega$. By taking the limits $t \to -\infty$ and $t' \to +\infty$ for the TTP, the conventional transmission probability, $T_{\varepsilon_f,\varepsilon_i}$, can be easily obtained. The numerical study shows that $T_{\varepsilon_f,\varepsilon_i}$ has a rich spectrum and exhibits a simultaneous quenching behaviour; these results are similar to those obtained earlier by Wagner [2].

The outline of this paper is as follows. In section 2, the model is presented and Keldysh's nonequilibrium-Green-function method is used to derive the transient transmission probability $T_{\varepsilon_f,\varepsilon_i}(t',t)$. In section 3, we present numerical studies of the TTP, and discuss under what conditions the resonant behaviour can be developed. The properties of the conventional transmission probability $T_{\varepsilon_f,\varepsilon_i}$ are studied in section 4. On the basis of the properties of $T_{\varepsilon_f,\varepsilon_i}$, the condition for the occurrence of an electron–photon pump is also discussed in this section. A brief summary is presented in section 5.

2. Model and formulation

The system under consideration is a quantum dot coupled to two leads through two barriers, and can be described by the following Hamiltonian:

$$H(t) = H_{\mathrm{lead}}(t) + H_{\mathrm{dot}}(t) + H_T$$

where

$$\begin{aligned} H_{\mathrm{lead}}(t) &= \sum_{k\in L} \varepsilon_k(t) a_k^\dagger a_k + \sum_{p\in R} \varepsilon_p(t) b_p^\dagger b_p \\ H_{\mathrm{dot}}(t) &= \varepsilon_0(t) c_0^\dagger c_0 \\ H_T &= \sum_{k\in L} V_k a_k^\dagger c_0 + \sum_{p\in R} V_p b_p^\dagger c_0 + \mathrm{HC}. \end{aligned} \tag{1}$$

$H_{\text{lead}}(t)$ describes noninteracting electrons in the leads. $a_k^\dagger$ (a_k) and $b_p^\dagger$ (b_p) are the creation (annihilation) operators of the electron in the left-hand and the right-hand lead, respectively. $H_{\text{dot}}(t)$ models the quantum dot. For simplicity, we only consider a single state in the quantum dot, and neglect the intra-dot electron–electron Coulomb interaction. H_T denotes the tunnelling part, which is time independent. We assume that the time-dependent external fields only cause rigid shifts of the single-electron energies $\varepsilon_\alpha(t)$ (here $\alpha = 0, k, p$), and do not change their occupations (the adiabatic approximation) [14–16], and that the effect of the magnetic fields can be neglected. We separate $\varepsilon_\alpha(t)$ into two parts: $\varepsilon_\alpha(t) = \varepsilon_\alpha + \Delta_\beta(t)$, where $\beta = \mathrm{L}, \mathrm{R}, 0$ corresponds to the left-hand lead, the right-hand lead, and the dot, respectively, and ε_α stands for the time-independent single-electron energies without time-dependent external fields. $\Delta_\beta(t)$ is a time-dependent part arising from the external fields.

In the following we derive the general formula for the TTP, $T_{\varepsilon_f,\varepsilon_i}(t', t)$, by using the nonequilibrium-Green-function technique. A wave packet incident from the noninteracting states in the left-hand lead at time t can be described as [13]

$$\phi(\varepsilon_i, t) = \sum_k \frac{1}{\sqrt{\rho_L(\varepsilon_k)\,\Delta\varepsilon}} \varphi\left(\frac{\varepsilon_k - \varepsilon_i}{\Delta\varepsilon}\right)|k, t\rangle \tag{2}$$

where $\varphi(x) = 1$ for $-1/2 < x < 1/2$, $\varphi(x) = 0$ for $x > 1/2$ or $x < -1/2$; and $\rho_L(\varepsilon_k)$ is the density of states in the left-hand lead. The centre of the wave packet is located at ε_i with a small energy width $\Delta\varepsilon$, $\Delta\varepsilon < \omega$, and $\Delta\varepsilon < \Gamma$.

The TTP, $T_{\varepsilon_f,\varepsilon_i}(t', t)$, is defined as the probability that an electron with the energy ε_i incident from the left-hand lead at time t will transmit through the quantum dot into the right-hand lead, with the energy ε_f at time t', and can obviously be written as

$$\begin{aligned} T_{\varepsilon_f,\varepsilon_i}(t', t) &= \sum_p T_{p,\varepsilon_i}(t', t)\delta(\varepsilon_p - \varepsilon_f) \\ T_{p,\varepsilon_i}(t', t) &= \left|\theta(t' - t)\langle p, t'| \sum_k \frac{1}{\sqrt{\rho_L(\varepsilon_k)\,\Delta\varepsilon}} \varphi\left(\frac{\varepsilon_k - \varepsilon_i}{\Delta\varepsilon}\right)|k, t\rangle\right|^2. \end{aligned} \tag{3}$$

Clearly, $\theta(t' - t)\langle p, t'|k, t\rangle$ is related to the retarded Green function $G^r_{p,k}(t', t)$ by

$$G^r_{p,k}(t', t) \equiv -\mathrm{i}\theta(t' - t)\langle 0|\{b_p(t'), a_k^\dagger(t)\}|0\rangle = -\mathrm{i}\theta(t' - t)\langle p, t'|k, t\rangle \tag{4}$$

where $|0\rangle$ denotes the electronic vacuum. By using the Keldysh equation, one can easily find that

$$G^r_{p,k}(t', t) = \int \mathrm{d}t_1\ \mathrm{d}t_2\ V_p V_k^* g_p^r(t', t_1) G^r_{00}(t_1, t_2) g_k^r(t_2, t). \tag{5}$$

Here

$$g_\alpha^r(t', t) = -\mathrm{i}\theta(t' - t)\exp\left\{-\,\mathrm{i}\int_t^{t'} \mathrm{d}t_1\ \varepsilon_\alpha(t_1)\right\} \qquad (\alpha = k, p)$$

is the exact Green function of the electron in the left-hand or the right-hand leads without coupling between the leads and the dot, and $G^r_{00}(t_1, t_2) \equiv -\mathrm{i}\theta(t_1 - t_2)\langle 0|\{c_0(t_1), c_0^\dagger(t_2)\}|0\rangle$. Substituting equation (5) into equation (3), the sum over k (or p), $\sum_k$ (or $\sum_p$), can be changed into an integral with the help of the density of states in the left-hand (or right-hand) lead, $\int \mathrm{d}\varepsilon\ \rho_L(\varepsilon)$ (or $\int \mathrm{d}\varepsilon\ \rho_R(\varepsilon)$). Then the TTP $T_{\varepsilon_f,\varepsilon_i}(t', t)$ becomes

$$\begin{aligned} T_{\varepsilon_f,\varepsilon_i}(t', t) = \frac{1}{\Delta\varepsilon} \int_{\varepsilon_i - \Delta\varepsilon/2}^{\varepsilon_i + \Delta\varepsilon/2} \mathrm{d}\varepsilon_{k_1} \int_{\varepsilon_i - \Delta\varepsilon/2}^{\varepsilon_i + \Delta\varepsilon/2} \mathrm{d}\varepsilon_k \int_t^{t'} \frac{\mathrm{d}t_1\ \mathrm{d}t_2\ \mathrm{d}s_1\ \mathrm{d}s_2}{(2\pi)^2}\ \Gamma_R(\varepsilon_f, t_1, s_1) \\ \times\ \Gamma_L(\varepsilon_i, s_2, t_2) G^r_{00}(t_1, t_2) G^{r*}_{00}(s_1, s_2) \mathrm{e}^{-\mathrm{i}[\varepsilon_f(s_1 - t_1) + \varepsilon_k t_2 - \varepsilon_{k_1} s_2]} \end{aligned} \tag{6}$$

and the generalized linewidth function $\Gamma_\alpha(\varepsilon, t, s)$ is defined as

$$\Gamma_\alpha(\varepsilon, t, s) = 2\pi\rho_\alpha(\varepsilon)V(\varepsilon)V^*(\varepsilon)\exp\left\{-\mathrm{i}\int_t^s \mathrm{d}t\ \Delta_\alpha(t)\right\} \tag{7}$$

where $\alpha = \mathrm{L}, \mathrm{R}$, $V(\varepsilon_k) = V_k$ and $V(\varepsilon_p) = V_p$.

In the following we make the wide-band-limit (WBL) approximation [13–17], in which the linewidth

$$\Gamma_\alpha(\varepsilon) = 2\pi\rho_\alpha(\varepsilon)V(\varepsilon)V^*(\varepsilon) \qquad (\alpha = \mathrm{L}, \mathrm{R})$$

is an energy-independent constant. The WBL approximation is widely used in mesoscopic transport problems, and using this approximation is justified under the following conditions:

(i) the bandwidth of the leads is much larger than the linewidth $\Gamma_\alpha(\varepsilon)$;

(ii) the density of states ($\rho_\alpha(\varepsilon)$ ($\alpha = \mathrm{L}, \mathrm{R}$)) and the hopping matrix elements ($V(\varepsilon_k)$ and $V(\varepsilon_p)$) vary slowly with energy over a range of several Γ around ε_0;

(iii) the energy level of the quantum dot, ε_0, is not close to the band bottom of the leads.

Under the WBL approximation and by using Dyson's equation, the retarded Green function $G^r_{00}(t_1, t_2)$ can be obtained as

$$G^r_{00}(t_1, t_2) = -\mathrm{i}\theta(t_1 - t_2)\exp\left\{-\mathrm{i}\int_{t_2}^{t_1} \varepsilon_0(t)\ \mathrm{d}t - \frac{\Gamma}{2}(t_1 - t_2)\right\} \tag{8}$$

where $\Gamma = \Gamma_L + \Gamma_R$. Substituting the expression for $G^r_{00}(t_1, t_2)$ into equation (6), and considering only the harmonic external fields, i.e. $\Delta_\alpha(t) = \Delta_\alpha\cos\omega t$ ($\alpha = \mathrm{L}, \mathrm{R}, 0$), the TTP $T_{\varepsilon_f,\varepsilon_i}(t', t)$ can be reduced to

$$\begin{aligned} T_{\varepsilon_f,\varepsilon_i}(t', t) = \frac{\Gamma_L\Gamma_R}{(2\pi)^2\,\Delta\varepsilon}\Bigg| &\int_{\varepsilon_i-\Delta\varepsilon/2}^{\varepsilon_i+\Delta\varepsilon/2} \mathrm{d}\varepsilon_k \int_t^{t'} \mathrm{d}t_1 \int_t^{t_1} \mathrm{d}t_2\ \exp\left\{\mathrm{i}\left(\varepsilon_f + \mathrm{i}\frac{\Gamma}{2} - \varepsilon_0\right)t_1\right. \\ &\left. + \mathrm{i}\frac{\Delta_R - \Delta_0}{\omega}\sin\omega t_1\right\}\exp\left\{\left(-\mathrm{i}\varepsilon_k + \frac{\Gamma}{2} + \mathrm{i}\varepsilon_0\right)t_2 - \mathrm{i}\frac{\Delta_L - \Delta_0}{\omega}\sin\omega t_2\right\}\Bigg|^2. \end{aligned} \tag{9}$$

By using the identity

$$\exp\{\mathrm{i}a\sin x\} = \sum_k J_k(a)\mathrm{e}^{\mathrm{i}kx}$$

and carrying out the integration over t_1, t_2, we obtain

$$\begin{aligned} T_{\varepsilon_f,\varepsilon_i}(t', t) = \frac{\Gamma_L\Gamma_R}{(2\pi)^2\,\Delta\varepsilon}\Bigg| &\sum_{n,m} J_n\left(\frac{\Delta_R - \Delta_0}{\omega}\right)J_m\left(\frac{\Delta_L - \Delta_0}{\omega}\right) \\ &\times\int_{\varepsilon_i-\Delta\varepsilon/2}^{\varepsilon_i+\Delta\varepsilon/2} \frac{\mathrm{d}\varepsilon_k}{b}\left\{\frac{\mathrm{e}^{at'} - \mathrm{e}^{at}}{a} - \frac{\mathrm{e}^{bt+ct'} - \mathrm{e}^{at}}{c}\right\}\Bigg|^2 \end{aligned} \tag{10}$$

where

$$\begin{aligned} a &= \mathrm{i}(\varepsilon_f - \varepsilon_k + n\omega - m\omega) \\ b &= \mathrm{i}\left(-\varepsilon_k + \varepsilon_0 - m\omega - \mathrm{i}\frac{\Gamma}{2}\right) \\ c &= \mathrm{i}\left(\varepsilon_f - \varepsilon_0 + n\omega + \mathrm{i}\frac{\Gamma}{2}\right). \end{aligned} \tag{11}$$

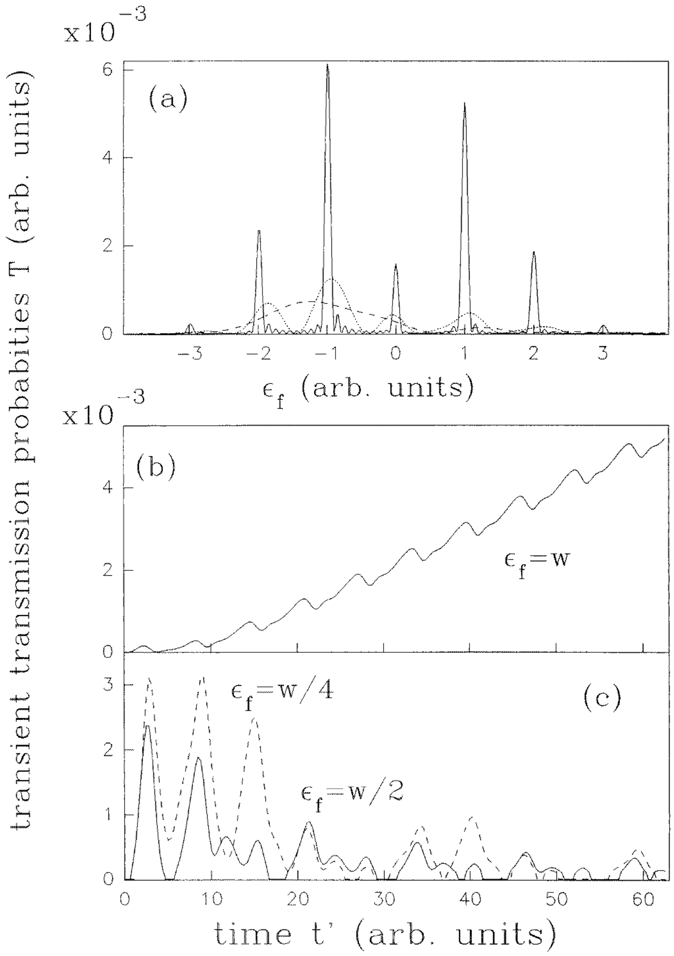

Figure 1. (a) $T_{\varepsilon_f,\varepsilon_i}(t',t)$ versus ε_f for varying t'. The solid, dotted, and dashed curves correspond to $t' = 10T$, $2T$, and T, respectively. (b) $T_{\varepsilon_f,\varepsilon_i}(t',t)$ versus t' for $\varepsilon_f = \omega$. (c) $T_{\varepsilon_f,\varepsilon_i}(t',t)$ versus t' for $\varepsilon_f = \omega/4$ (dashed curve) and $\varepsilon_f = \omega/2$ (solid curve), respectively. Other parameters: $t = 0$, $\varepsilon_i = \varepsilon_0 = 0$, $\Gamma_L = \Gamma_R = 0.2$, $\omega = 1$, $\Delta_0 = 2$, and $\Delta_R = \Delta_L = 0$. The numbers on the axis for $T_{\varepsilon_f,\varepsilon_i}(t',t)$ in (c) are in units of 10^{-4}.

Similarly, we can obtain the transient reflection probability $R_{\varepsilon_r,\varepsilon_i}(t',t)$ (not shown here). One should notice that

$$\int T_{\varepsilon_f,\varepsilon_i}(t',t)\ \mathrm{d}\varepsilon_f + \int R_{\varepsilon_r,\varepsilon_i}(t',t)\ \mathrm{d}\varepsilon_r$$

is not equal to 1 because the occupation number $n(t)$ of the quantum dot depends on the time t.

3. The transient transmission probability

In order to understand the detailed properties of the TTP, we need to perform numerical studies. From equation (10) one easily finds that $T_{\varepsilon_f,\varepsilon_i}(t',t)$ depends on the energy width of

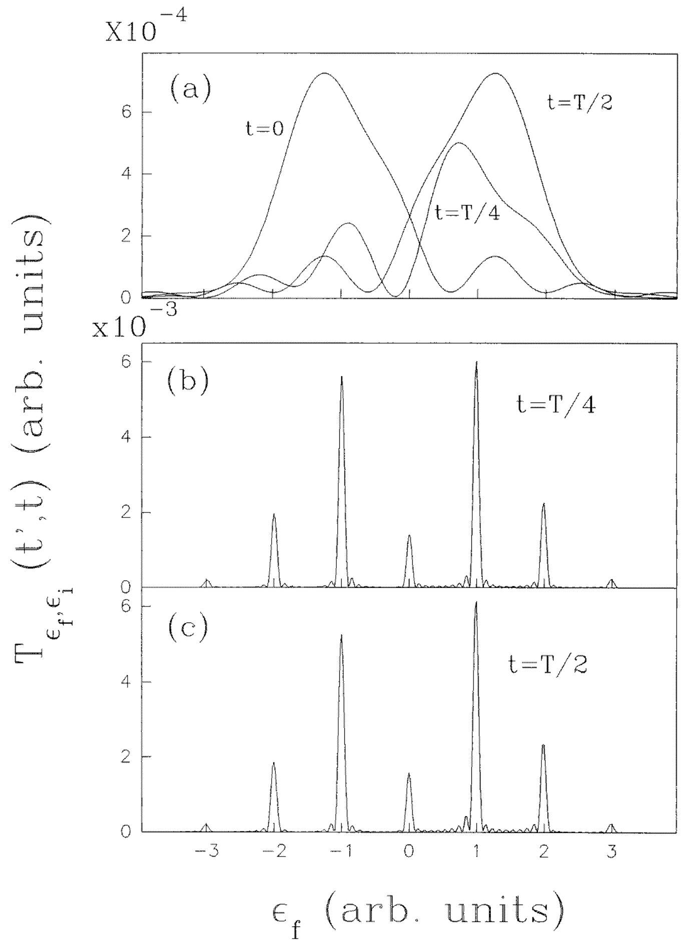

Figure 2. The TTP $T_{\varepsilon_f,\varepsilon_i}(t', t)$ versus ε_f at different t, with different values of $t' - t$, and with $\varepsilon_i = \varepsilon_0 = 0$, $\Gamma_L = \Gamma_R = 0.2$, $\Delta_0 = 2$, $\Delta_L = \Delta_R = 0$. (a) $t' - t = T$; the three solid curves correspond to $t = 0$, $T/4$, and $T/2$. (b) $t' - t = 10T$; $t = T/4$. (c) $t' - t = 10T$; $t = T/2$. The case with $t = 0$ has been shown in figure 1(a).

the incident wave packet, $\Delta\varepsilon$. The time taken for the electron to tunnel through the dot is about $1/\Delta\varepsilon$. If $\Delta\varepsilon \to 0$, the time will approach ∞. In our calculation, to avoid the effect of $\Delta\varepsilon$, we set $\Delta\varepsilon(t' - t)$ at the constant value 2π. For $t' - t = T$ ($T = 2\pi/\omega$ is the period of the external field), $\Delta\varepsilon \sim 0.025\omega$, which satisfies the condition $\Delta\varepsilon < \omega$ perfectly.

Figure 1(a) shows $T_{\varepsilon_f,\varepsilon_i}(t', t)$ versus ε_f for different times t', for a fixed incidence time t. For a small time difference, $t' - t = T \sim 1/\Gamma$, a visible proportion of the electron transmission probability will be distributed among the energies for which $\varepsilon_f - \varepsilon_i \neq n\omega$ ($n = 0, \pm 1, \ldots$), and no sharp peak emerges. For $t' - t = 10T \gg 1/\Gamma$, a series of sharp peaks located at the energies for which $\varepsilon_f - \varepsilon_i = n\omega$ ($n = 0, \pm 1, \ldots$) emerge while the transmission probability is almost zero for the other energies, i.e. the transmission probability exhibits a striking resonant behaviour. In fact, when $t' - t \to +\infty$, the peaks get higher and narrower, and finally become a series of δ-functions. Figures 1(b) and 1(c) show $T_{\varepsilon_f,\varepsilon_i}(t', t)$ versus t' (with $t = 0$) for different energies ε_f of the electron leaving the dot. For $\varepsilon_f - \varepsilon_i = n\omega$, as in figure 1(b), $T_{\varepsilon_f,\varepsilon_i}(t', t)$ exhibits an oscillatory increase with the increase of the time t' at which the electron leaves the dot. However, for $\varepsilon_f - \varepsilon_i \neq n\omega$ (figure 1(c)), $T_{\varepsilon_f,\varepsilon_i}(t', t)$ tends to show a decrease in the magnitude of the oscillation with the increase of t'. The patterns of the oscillation are different for different incidence times t (not shown here), but all of the oscillation will gradually disappear as the time $t' \to +\infty$.

Figure 2 shows $T_{\varepsilon_f,\varepsilon_i}(t', t)$ versus ε_f for different incidence times t for certain time differences $t' - t$. For $t' - t = T \sim 1/\Gamma$, $T_{\varepsilon_f,\varepsilon_i}(t', t)$ versus ε_f is strongly dependent on the incidence time t (figure 2(a)). On the other hand, for $t' - t = 10T \gg 1/\Gamma$, almost no visible difference between the cases of $t = T/4$ (figure 2(b)) and $t = T/2$ (figure 2(c)) can be seen.

All of these properties can be understood as follows. For large enough time difference, $t' - t \gg 1/\Gamma$, an electron can tunnel through the system by many different processes: an electron can first tunnel from the left-hand lead into the dot through the left-hand barrier then directly tunnel through the right-hand barrier to the right-hand lead; or, after tunnelling into the dot, the electron can travel in the dot and be reflected twice (four times, six times, ..., etc) by the left-hand and right-hand barrier, and then tunnel to the right-hand lead. In fact, the TTP $T_{\varepsilon_f,\varepsilon_i}(t', t)$ is the sum of the contributions from all of these processes. Because of the coherent superposition, the resonant behaviour is well developed. Again since $\omega > \Gamma$ (i.e. $1/\Gamma > T/2\pi$), photon-assisted resonant tunnelling emerges too [12, 17]. On the other hand, for a small time difference, say $t' - t \sim 1/\Gamma$, an electron tunnels through the system mainly by the direct tunnelling process, so the tunnelling does not exhibit visible resonant behaviour. One should note that the condition $t' - t \gg 1/\Gamma$ for well developed resonant tunnelling will constrain the upper limit of the response time τ for resonant tunnelling devices. The larger Γ, the shorter τ. If $\Gamma = 1$ meV [18], the response time $\tau \sim 10^{-11}$ s, and $\Gamma = 0.05$ eV, [19], $\tau \sim 2 \times 10^{-13}$ s.

4. The conventional transmission probability

By definition, the conventional transmission probability, $T_{\varepsilon_f,\varepsilon_i}$, is the limit of the TTP (equation (10)):

$$
\begin{aligned}
T_{\varepsilon_f,\varepsilon_i} &\equiv \lim_{\Delta\varepsilon\to 0} \lim_{\substack{t'\to+\infty \\ t\to-\infty}} T_{\varepsilon_f,\varepsilon_i}(t', t) \\
&= \sum_l \Gamma_L \Gamma_R \left| \sum_n \left[J_n\left(\frac{\Delta_R - \Delta_0}{\omega}\right) J_{n-l}\left(\frac{\Delta_L - \Delta_0}{\omega}\right)\right] \right. \\
&\quad \times \left. \left(\varepsilon_i - \varepsilon_0 + n\omega - l\omega + \mathrm{i}\frac{\Gamma}{2}\right)^{-1} \right|^2 \delta(\varepsilon_f - \varepsilon_i + l\omega). \qquad (12)
\end{aligned}
$$

Similarly one can obtain the reflection probability $R_{\varepsilon_r,\varepsilon_i}$ as

$$
\begin{aligned}
R_{\varepsilon_r,\varepsilon_i} &= \delta(\varepsilon_r - \varepsilon_i)\left[1 - \sum_m J_m^2\left(\frac{\Delta_0 - \Delta_L}{\omega}\right)\Gamma_L\Gamma \Big/ \left((\varepsilon_i - \varepsilon_0 - m\omega)^2 + \frac{\Gamma^2}{4}\right)\right] \\
&\quad + \sum_l \Gamma_L^2 \delta(\varepsilon_r - \varepsilon_i - l\omega) \\
&\quad \times \left| \sum_n \left[J_n\left(\frac{\Delta_0 - \Delta_L}{\omega}\right) J_{n-l}\left(\frac{\Delta_0 - \Delta_L}{\omega}\right) \Big/ \left(\varepsilon_i - \varepsilon_0 + l\omega - n\omega + \mathrm{i}\frac{\Gamma}{2}\right)\right]\right|^2. \qquad (13)
\end{aligned}
$$

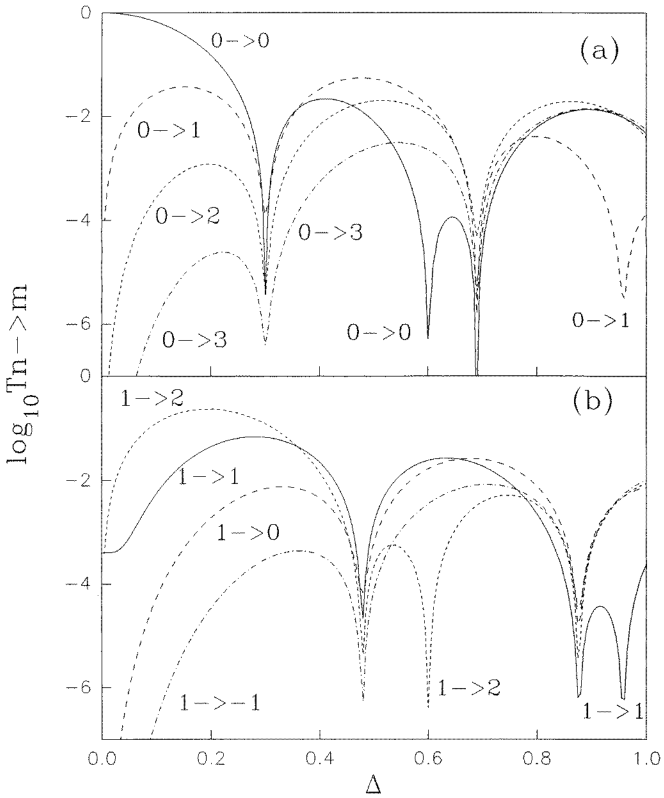

Figure 3. $T_{n\to m}$ versus Δ, with $\omega = 1$, $\Gamma_L = \Gamma_R = 0.02$, $\varepsilon_0 = 0$, $\Delta_0 = 0$, $\Delta_L/\Delta_{Lmax} = \Delta$, $\Delta_R/\Delta_{Rmax} = \Delta$, and $\Delta_{Lmax} = 8$, $\Delta_{Rmax} = 4$. (a) For $n = 0$, (b) for $n = 1$.

It can be proved easily that

$$\int \mathrm{d}\varepsilon\ (T_{\varepsilon,\varepsilon_i} + R_{\varepsilon,\varepsilon_i})$$

is exactly equal to 1.

Comparing with the case without external fields, the transmission probability exhibits many extra tunnelling channels originating from the absorption or emission of one or multiple photons. We introduce $T_{n\to m}$ to describe the transmission probability for an electron incident with the energy $\varepsilon_i = n\omega + \varepsilon_0$ and leaving with the energy $\varepsilon_f = m\omega + \varepsilon_0$. From our numerical studies, we find the following properties of $T_{n\to m}$.

(i) If the harmonic external field is only applied to the quantum dot, then one reproduces the previously obtained results for $T_{n\to m}$ versus Δ_0/ω (not shown here) obtained by Wagner (shown in figure 4(b) and figure 5(b) in reference [2]).

(ii) For asymmetric harmonic external fields applied to the left-hand and right-hand leads, i.e. $\Delta_R \neq \Delta_L$, $T_{n\to m}$ versus Δ_α/ω ($\alpha = \mathrm{L, R}$) is as given in figure 3. Like in case (i), $T_{n\to m}$ does not increase with Δ_α monotonically, but instead has a very rich spectrum depending on Δ_α/ω, n, and m. In particular, it shows a simultaneous quenching for a certain group of transmission channels. The difference from case (i) lies in the shapes of the curves: the amplitude corresponding to $n \to m$ is approximately proportional to

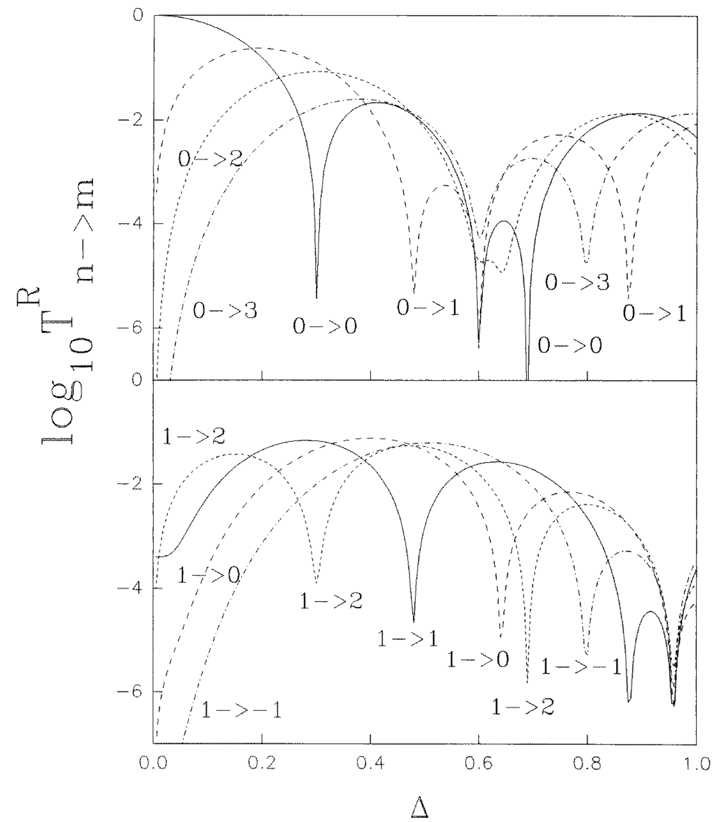

Figure 4. $T^R_{n\to m}$ versus Δ, with the same parameters as for figure 3.

$|J_n(\Delta_L/\omega)J_m(\Delta_R/\omega)|^2$, and the locations of simultaneous quenching are at the zeros of either $J_n(\Delta_L/\omega)$ or $J_m(\Delta_R/\omega)$.

(iii) Let T^R denote the probability of transmission with an electron coming in from the right-hand lead and going out to the left. T^R is easily obtained by exchanging L and R in equation (12). Figure 4 shows $T^R_{n\to m}$ versus Δ_L, Δ_R with the same parameters as for figure 3. The simultaneous quenching and the rich spectrum dependence on Δ_α/ω ($\alpha = \mathrm{L, R}$), n, and m are similar to the features of $T^L_{n\to m}$ (i.e. $T_{n\to m}$), but the shapes of the curves change significantly.

Recently, electron–photon pump phenomena have been observed and studied [8, 9, 20]. A criterion for the occurrence of an electron–photon pump can be derived from the difference between T^R and T^L. By using $T^L_{\varepsilon_f,\varepsilon_i}$ and $T^R_{\varepsilon_f,\varepsilon_i}$, the averaged current $\langle j\rangle$ can be expressed as

$$\langle j(t)\rangle = e\int \mathrm{d}\varepsilon_i\ \mathrm{d}\varepsilon_f\ T^L_{\varepsilon_f,\varepsilon_i} f^L(\varepsilon_i)\left[1-f^R(\varepsilon_f)\right] - e\int \mathrm{d}\varepsilon_i\ \mathrm{d}\varepsilon_f\ T^R_{\varepsilon_f,\varepsilon_i} f^R(\varepsilon_i)\left[1-f^L(\varepsilon_f)\right]. \tag{14}$$

When the bias is zero and the temperature of the left-hand lead is equal to that of the

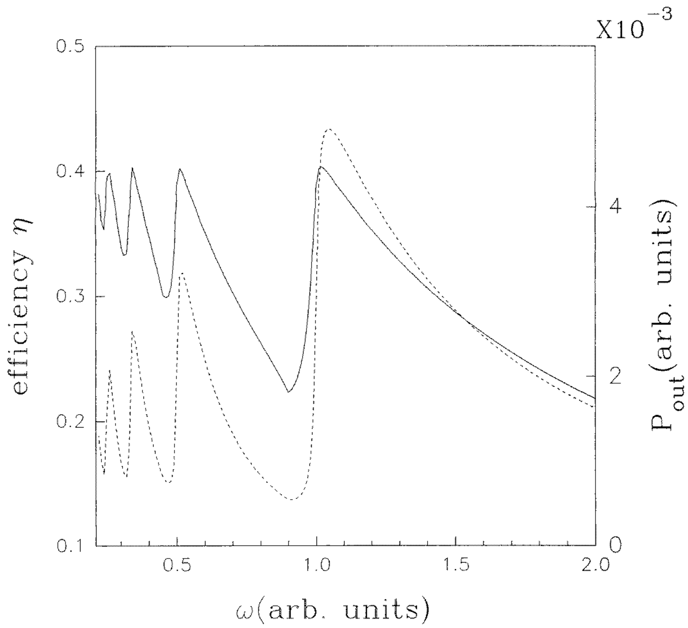

Figure 5. The dependence of the pumping efficiency η (solid line) and the output power P_{out} (dotted line) on the frequency ω. The other parameters are $\Delta_0 = \Delta_R = 0$, $\Delta_L = 1$, $\varepsilon_0 = 1$, $\Gamma_L = \Gamma_R = 0.01$, and the bias $v = 0.9$ ($\mu_L = 0$, $\mu_R = 0.9$), and the temperature of each lead is zero.

right-hand lead, the current $\langle j(t)\rangle$ reduces to

$$\langle j(t)\rangle = e\int \mathrm{d}\varepsilon_i\ \mathrm{d}\varepsilon_f\ \left[T^L_{\varepsilon_f,\varepsilon_i} - T^R_{\varepsilon_f,\varepsilon_i}\right] f^L(\varepsilon_i)\left[1 - f^R(\varepsilon_f)\right]. \tag{15}$$

Obviously, the electron–photon pump cannot be obtained if $T^L_{\varepsilon_f,\varepsilon_i} = T^R_{\varepsilon_f,\varepsilon_i}$. On the other hand, once $T^L_{\varepsilon_f,\varepsilon_i} \neq T^R_{\varepsilon_f,\varepsilon_i}$, the pumping phenomenon must emerge at a certain gate voltage. The pumping efficiency η is defined as $\eta = \langle j(t)\rangle v/P_{\mathrm{in}}$ [20], where v is the bias and P_{in} is the input power. Assuming that the external field only exchanges energy with the electronic system, and making the adiabatic approximation, i.e. assuming that the external field does not change the electronic distribution function in the same region, the input power P_{in} can be obtained as

$$P_{\mathrm{in}} = \int \mathrm{d}\varepsilon \int \mathrm{d}\varepsilon_i\ (\varepsilon - \varepsilon_i)\left[T^L_{\varepsilon,\varepsilon_i} f^L(\varepsilon_i)(1 - f^R(\varepsilon)) + T^R_{\varepsilon,\varepsilon_i} f^R(\varepsilon_i)(1 - f^L(\varepsilon)) + R^L_{\varepsilon,\varepsilon_i} f^L(\varepsilon_i)(1 - f^L(\varepsilon)) + R^R_{\varepsilon,\varepsilon_i} f^R(\varepsilon_i)(1 - f^R(\varepsilon))\right]. \tag{16}$$

Figure 5 shows the dependences of the pumping efficiency η and the output power P_{out} on the frequency ω of the external field coupled only to the left-hand lead. Both η and P_{out} exhibit a series of peaks at $\omega = \omega_n \equiv (\varepsilon_0 - \mu)/n$ ($n = 1, 2, 3, \ldots$), and η drops sharply, while ω is slightly smaller than ω_n. From peak to peak, η shows almost no change, but P_{out} gradually gets smaller. The reason for this is as follows. When $\omega = \omega_n$, the external field pumps the electron from the Fermi surface of the lead to the dot state ε_0 with the help of the n-photon process. While ω is slightly smaller than ω_n, the n-photon process will be suppressed, and the pumping is mainly caused by the $(n+1)$-photon process; then the external field pumps the electron from somewhere in the Fermi sea of the lead to the dot state with $\varepsilon = \varepsilon_0 - (n+1)\omega \simeq \mu_L - \omega$, consuming much energy and causing a sharp reduction of η.

5. Conclusions

In this work, we studied the electron tunnelling through a quantum dot system under the influence of oscillating external fields. The main goal was that of finding the time-dependent behaviour of the transient transmission probability (TTP), $T_{\varepsilon_f,\varepsilon_i}(t', t)$. The theoretical studies reveal that whether or not the resonant behaviour is well developed depends critically on whether or not $t' - t \gg 1/\Gamma$. We investigated the properties of the conventional transmission probability $T_{\varepsilon_f,\varepsilon_i}$, and obtained results similar to those obtained by Wagner, with some differences. The condition for an electron–photon pump to arise has also been discussed on the basis of our theoretical results for $T_{\varepsilon_f,\varepsilon_i}$.

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References

[1] Sokolovski D 1988 *Phys. Rev.* B **37** 4201
[2] Wagner M 1994 *Phys. Rev.* B **49** 16 544
[3] Wagner M 1996 *Phys. Rev. Lett.* **76** 4010
[4] Johansson P and Wendin G 1992 *Phys. Rev.* B **46** 1451
[5] Iñarrea J, Platero G and Tejedor C 1994 *Phys. Rev.* B **50** 4581
[6] Iñarrea J and Platero G 1995 *Phys. Rev.* B **51** 5244
[7] Yakubo K, Feng Shechao and Hu Qing 1996 *Phys. Rev.* B **54** 7987
[8] Kouwenhoven L P, Jauhar S, Orenstein J, McEuen P L, Nagamune Y, Motohisa J and Sakaki H 1994 *Phys. Rev. Lett.* **73** 3443
[9] Kouwenhoven L P, Jauhar S, McCormick K, Dixon D, McEuen P L, Nazarov Yu V, van der Vaart N C and Foxon C T 1994 *Phys. Rev.* B **50** 2019
[10] Blick R H, Haug R J, van der Weide D W, von Klitzing K and Eberl K 1995 *Appl. Phys. Lett.* **67** 3924
[11] Drexler H, Scott J S, Allen S J, Campman K L and Gossard A C 1995 *Appl. Phys. Lett.* **67** 2816
[12] Chitta V A, Kutter C, de Bekker R E M, Mann J C, Hawksworth S J, Chamberlain J M, Henini M and Hill G 1994 *J. Phys.: Condens. Matter* **6** 3945
[13] Wingreen N S, Jacobsen K W and Wilkins J W 1989 *Phys. Rev.* B **40** 11 834
[14] Wingreen N S, Jauho A-P and Meir Y 1993 *Phys. Rev.* B **48** 8487
[15] Jauho A-P, Wingreen N S and Meir Y 1994 *Phys. Rev.* B **50** 5528
[16] Sun Qing-feng and Lin Tsung-han 1997 *J. Phys.: Condens. Matter* **9** 4875
[17] Sun Qing-feng and Lin Tsung-han 1997 *J. Phys.: Condens. Matter* **9** 3043
[18] Meirav U, Kastner M A and Wind S J 1990 *Phys. Rev. Lett.* **65** 771
[19] Sollner T C L G, Goodhue W D, Tannenwald P E, Parker C D and Peck D D 1983 *Appl. Phys. Lett.* **43** 588
[20] Stafford C A and Wingreen N S 1996 *Phys. Rev. Lett.* **76** 1916